

Field and Theoretical Analysis of Accelerated Consolidation Using Vertical Drains

Chinmay Joshi¹, Dr. A. R. Katti²

¹ Post Graduate Student ² Professor

^{1,2}Department of Civil Engineering, Datta Meghe College of Engineering, Navi Mumbai, India

Abstract: Mumbai is the region consisting of soft compressible marine clay deposits. There are several construction problems on such soils and thus ground improvement is need to be carried out. Vertical drains is generally preferred technique as accelerated settlement is achieved during the construction phase itself if planned accordingly. The concept of vertical drains is based on the theory of three dimensional consolidation as described by Terzaghi (1943). Based on this concept, a consolidation programme is developed and an attempt is made to determine the field to laboratory coefficient of vertical consolidation ratio by Taylor's Square Root of Time Method and Casagrande's Logarithm of Time Fitting Method for this region by considering the case study of MRVC at Virar Embankment. Based on this ratio, the rate of consolidation and time required for consolidation in the field can be determined knowing the consolidation parameters. Equations are developed by using output of the programme and it is explained.

Keywords: Soft Compressible Clay, Vertical Drains, Consolidation.

I. INTRODUCTION

In the early times before the advancement in the geotechnical engineering, the only alternate for the foundation engineers was to design the foundation matching to the sub-soil conditions at the provided site. But now a days, due to the advancement in geotechnical techniques and with the help of latest technology it is possible for us to alter the engineering characteristics of weak founding soil to suit the foundation of our choice. This geotechnical processes of improving the quality of the founding soil to our desired requirements are called as 'Ground Improving'.

In case of highly compressible saturated soft clay, imposition of load generates excess pore water pressure in soft layer. This excess pore water pressure may trigger both shear and settlement failures if not monitored and altered. This paper presents analysis and monitoring of ground improvement of soft saturated marine clays.

II. LITERATURE REVIEW

A. Brief Review:

To address settlement issues, literature review has been carried out for the theories related to three dimensional consolidation, and methods related to evaluation of consolidation parameters.

Terzaghi (1943), proposed one dimensional consolidation model and developed the corresponding analytical solution to explain, its mechanism and the phenomenon of the settlement of soil under surcharge, which triggered the study of the consolidation theory. Terzaghi, proposed piston and spring analogy for understanding the process of consolidation.

The basic differential equation proposed by Terzaghi is:

$$\frac{\partial u}{\partial t} = \frac{k}{\gamma_w m_v} \frac{\partial^2 u}{\partial z^2} = c_{vz} \frac{\partial^2 u}{\partial z^2}$$

Where 'c_{vz}' is coefficient of vertical consolidation, 'k' is the coefficient of permeability, 'γ_w' is the unit weight of water and 'm_v' is coefficient of volume change:

$$c_{vz} = \frac{k}{\gamma_w m_v}$$

The solution for the above differential equation can be obtained by considering proper boundary conditions and by solving Fourier series as:

$$u = \sum_{N=0}^{N=\infty} \frac{2\Delta p}{m} \left[\sin \frac{mz}{H} \right] e^{(-m^2 c_{vz} t)/H^2}$$

Wherein, 'm' is an integer, 't' is time, 'H' is the thickness of the clay layer, 'Δp' is increment in pressure and z gives the variation in depth.

To arrive at a solution, use of two non-dimensional parameters are introduced. The first non-dimensional group is the time factor T_v where:

$$T_v = \frac{c_{vz} * t}{H^2}$$

The second non-dimensional group is the degree of consolidation 'U'. The term 'U' is expressed as the ratio of the amount of consolidation which has already taken place to the total amount which is to take place under the load increment and is represented as:

$$U\% = 100 \left(1 - \sum_{N=0}^{N=\infty} \frac{2}{m^2} e^{-m^2 T_v} \right)$$

For the values of U% between 0 and 52.6%, T_v can be represented as:

$$T_v = \frac{\pi}{4} \left(\frac{U\%}{100} \right)^2$$

For the values of U% greater than 52.6%, T_v can be represented as:

$$T_v = 1.781 - 0.933 \log(100 - U\%)$$

Barron (1948), presented an analytical solution for combined vertical and radial drainage by decoupling the radial and vertical drainage at first and then attaining a product of the contribution from the radial and vertical drainage. Formulas for consolidation by vertical and radial flow to wells, for free strain and equal strain with or without peripheral smear and drain well resistance were also analyzed.

The differential equation for consolidation for equal strain case without smear and well resistance is given as:

$$\frac{\partial \bar{u}}{\partial t} = c_h \left(\frac{1}{r} \frac{\partial \bar{u}}{\partial r} + \frac{\partial^2 \bar{u}}{\partial r^2} \right) + c_{vz} \frac{\partial^2 \bar{u}}{\partial z^2}$$

Wherein, 'c_h' is the co-efficient of consolidation for horizontal flow, 'ū' is excess pore water pressure and 'r' is radial distance.

For radial flow only, 'c_{vz}' will be zero.

A solution for this second order expression is:

$$u_r = \frac{4\bar{u}}{de^2 * F(n)} \left[re^2 * \ln \left(\frac{r}{r_w} \right) - \frac{r^2 - r_w^2}{2} \right]$$

In which,

$$\bar{u} = u_0 e^\lambda$$

Wherein, 'e' is the base of natural logarithm,

$$\lambda = \frac{-8T_h}{F(n)}$$

And

$$F(n) = \frac{n^2}{n^2 - 1} \ln(n) - \frac{3n^2 - 1}{4n^2}$$

Whereas the solution for same differential equation for equal strain case with smear zone at periphery is:

$$u_r = \bar{u}_r \frac{\left[\ln \left(\frac{r}{r_s} \right) - \frac{r^2 - r_s^2}{2r_s^2} + \frac{k_h}{k_s} \left(\frac{n^2 - s^2}{n^2} \right) \ln(s) \right]}{v}$$

In which,

$$v = F(n, S, k_h, k_s)$$

$$m = \frac{k_h}{k_s} \left(\frac{n^2 - S^2}{n^2} \right) \ln(S) - \frac{3}{4} + \frac{S^2}{4n^2} + \frac{n^2}{n^2 - S^2} \ln\left(\frac{n}{S}\right)$$

And,

$$\bar{u}_r = u_0 \varepsilon^\xi$$

In which,

$$\xi = \frac{-8T_h}{m}$$

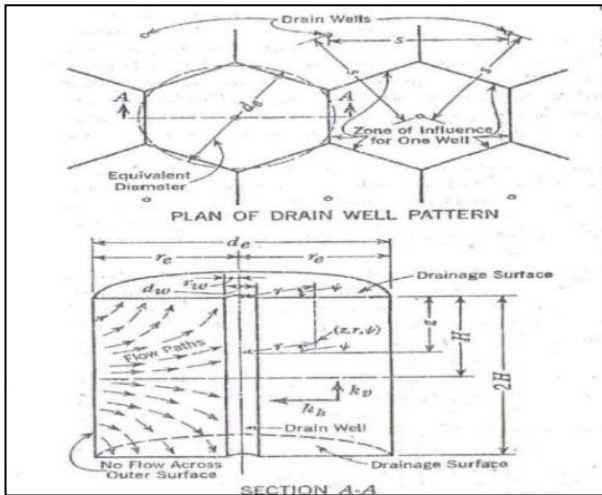


Fig. 1 Plan of drain well pattern and fundamental concepts of flow within zone of influence of each well

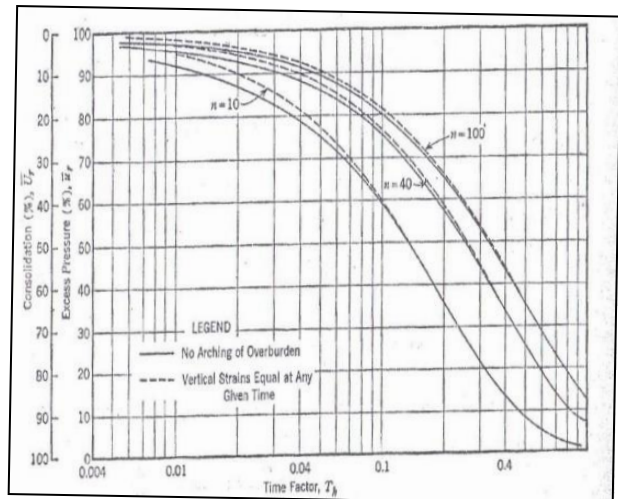


Fig. 2 Average degree of consolidation for various values of 'n' under 'equal strain' condition at any given time

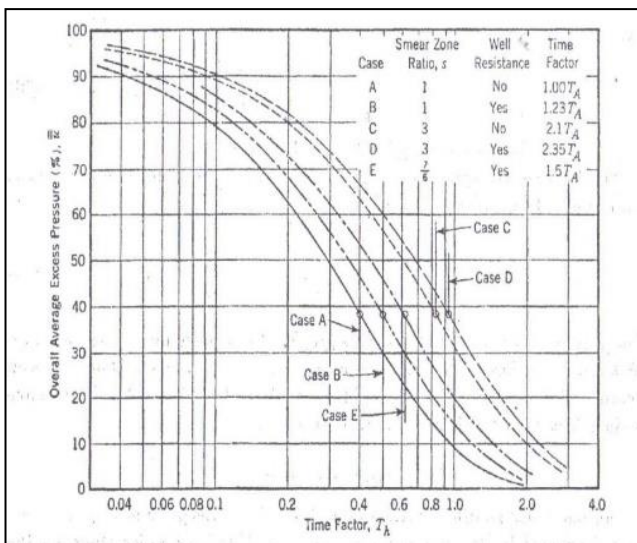


Fig. 3 Effect of smear and well resistance on 'equal strain' consolidation by radial flow to drain wells

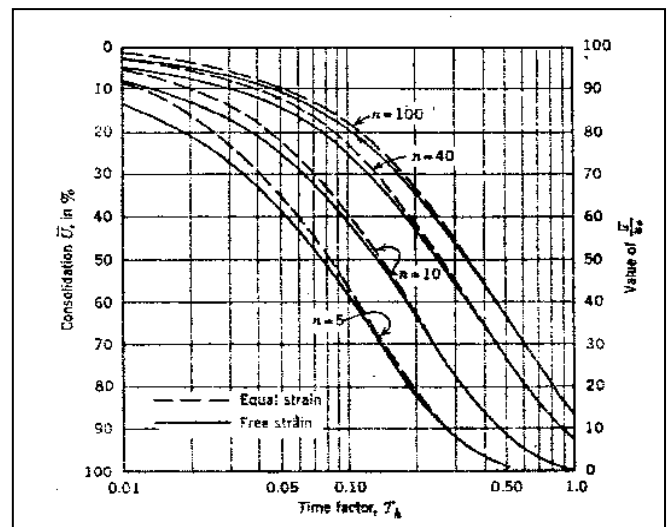


Fig. 4 Comparison of equal strain and free strain

Biot (1941), extended the classical reviews of Terzaghi's one dimensional problem of column under a constant load to three dimensional case and established equations valid for any arbitrary load variable with time. In this theory, Biot interpreted the mathematical formulation of the physical properties of soil and number of constants used to describe this property. Johnson (1970), gave the detailed use of vertical drains as a pre-compression technique for improving the properties of compressible soils. Richart (1959), presented diagrams for quantitative evaluation of equivalent "ideal well" of reduced diameter. The theories for consolidation due to vertical flow and radial flow of water to drain well was also reviewed. Hansbo (1979), made extensive sand drain study involving large scale field tests and observations of sand drain in soft clays. The consolidation process of clay by band shaped prefabricated drains was also studied and considered the design considerations.

Various case records for 'c_{vz} (field)/c_{vz} (lab)' ratio have also been recorded for vertical drains by different methods. Bergado (1991), studied the effectiveness of Mebra prefabricated drains inside the AIT campus by constructing 4m high embankment. Bergado (1991), analysed time-settlement data for Bangna-Bangpakong highway and the coefficient of

consolidation 'cvz' was back-figured from the field performance of the highway embankment and the following correlations was found 'c_{vz} (field)/c_{vz} (lab)' = 26. Leroueil (1987), showcased the 'c_{vz} (field)/c_{vz} (lab)' ratio for more than 15 sites. Dhowian, et. al. (1987), found the average ratio of field to laboratory 'cvz' to be in order of 55 for the Sabkha sediments.

B. Analysis:

As per Terzaghi's theory of one dimensional consolidation, it was assumed that the soil is laterally confined and the strains are in vertical direction only. In most of the actual problems surface loadings cause excess pore pressure which will vary both radially and vertically. The resulting consolidation will involve radial as well as vertical flow. Such a process is called 'Three Dimensional Consolidation'.

The basic differential three dimensional consolidation equation in polar coordinates can be expressed as:

$$c_{vr} \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} \right) + c_{vz} \frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t}$$

The general solution for the above equation can be given by the combination of the one dimensional flow and radial flow as:

$$(1 - U) = (1 - U_z) (1 - U_r)$$

Wherein, U = degree of consolidation for three dimensional flow

U_z = degree of consolidation for one dimensional flow (in vertical direction)

U_r = degree of consolidation for radial flow.

Programme developed for evaluation of consolidation:

Based on the procedures suggested by Barron (1948), rigorous analysis has been carried out to understand the behaviour of coefficient of consolidation with time for different 'k_v' (coefficient of permeability in vertical direction) and 'k_h' (coefficient of permeability in horizontal direction) parameters. For evaluation of consolidation, a programme is developed in which, basic parameters which are obtained from soil exploration programme, field and laboratory tests are used as input parameters.

In short, for the analysis of 11 m Depth of clay layer; by keeping depth of clay layer constant and using each 'c_h to c_v' ratio for analysis of 4 different ratios, we get the results for different centre to centre spacing of vertical drains, and for varying percent consolidation and time. This procedure is carried out for any depth of clay layer.

TABLE I: VARIOUS INPUT PARAMETERS FOR PROGRAMME

Description	Legends	Units	Value
Depth of clay layer	H	m	11.00
Bulk Density of clay layer	γ ₂	g/cm ³	1.40
Coefficient of consolidation in Vertical Direction	C _{vz}	cm ² /sec	4.00E-04
Relation between Cvz and Cvr			1
Coefficient of consolidation in Horizontal Direction	C _{vr}	cm ² /sec	4.00E-04
Height of working platform	H _{wp}	m	1.00
Density of working platform	γ ₁	g/cm ³	1.80
Height of embankment	H _e	m	2.00
Density of embankment material	γ ₃	g/cm ³	1.80
INPUT FOR TREATED LAYER			
Height of band drain	H _{sd}	m	10.50
Value of C _{vz}	C _{vz}	cm ² /sec	4.00E-04
Value of CVR	C _{vr}	cm ² /sec	4.00E-04
Drainage condition			Double as SD
Sand Drain Diameter	d	cm	6.50
Spacing of Sand Drain	s	m	0.50
		cm	50.00
Drain Layout			
Triangular	3		
Square	4		
Pattern of sand drain			3

INPUT FOR SMEAR ZONE			
Radius of Drain well	r_w	cm	3.25
Relation between r_w and r_s			1
Radius of Smear Zone	r_s	cm	3.25
Permeability of soil in horizontal direction	K_h		1.00
Relation between K_h and K_s			1
Permeability of smear zone	K_s		1.00
INPUT FOR UNTREATED LAYER			
Thickness of Untreated Clay Layer	H_{cl}	m	0.50
CV of Untreated clay layer	C_v	cm^2/sec	4.00E-04
Drainage Condition			
Single	1		
Double	2		
Drainage Condition			1

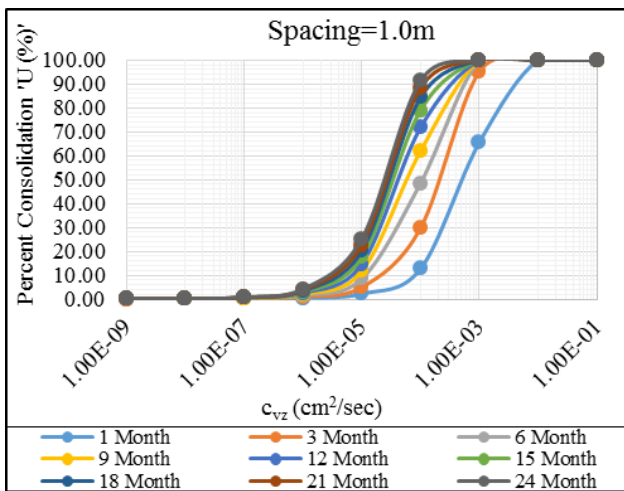


Fig. 5 Variation of coefficient of consolidation with percent consolidation for varying time, constant spacing ($S=1.0m$)

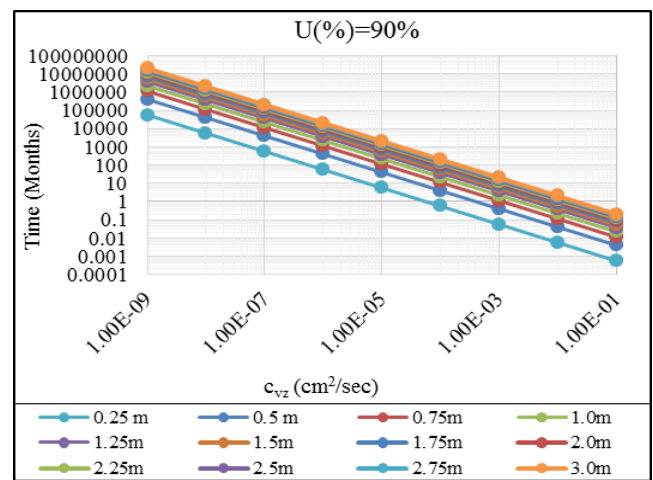


Fig. 6 Variation of coefficient of consolidation with time for varying spacing, constant percent consolidation ($U\%=90\%$)

Case Study: MRVC at Virar Embankment

This MRVC Project of Car Shed is located between Nallasopara and Virar stations of Western Railway. The alignment takes North-West to Mumbai. It is 20 kms. From Mumbai Entry Post at Dahisar and about 40 kms. From Andheri. The Google Earth Image of the project location as depicted in a Fig.7.



Fig. 7 Location of MRVC at Virar Embankment Site

TABLE II: TYPICAL SOIL PROPERTIES ADOPTED FOR DESIGN BASED LABORATORY AND FIELD TESTS

Sr. no	Soil Properties	Value	Unit
i.	Dry Density	1750	kg/m ³
ii.	Natural Moisture content	20-33	%
iii.	Specific Gravity	2.64	
iv.	Liquid Limit	55-90	%
v.	Plastic Limit	20-33	%
vi.	Undrained cohesion	2000-4000	kg/m ²
vii.	Compression index	0.29-0.50	
viii.	Coefficient of consolidation	2.30 x 10 ⁻⁰⁴	cm ² /sec

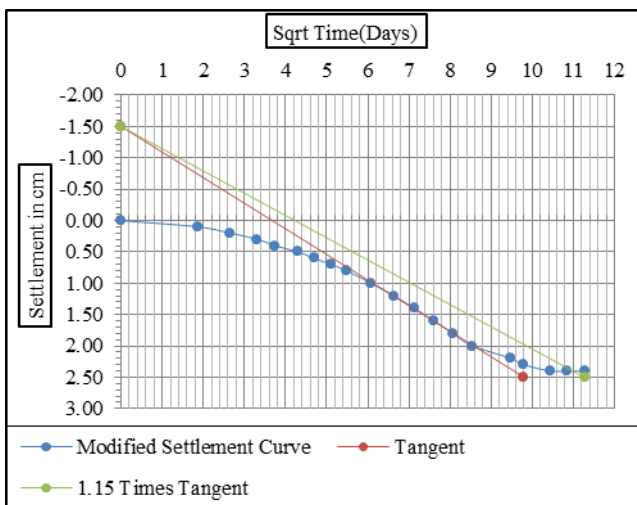


Fig. 8 Modified settlement curve/ Taylor's Method (Settlement marker) (CH-1)

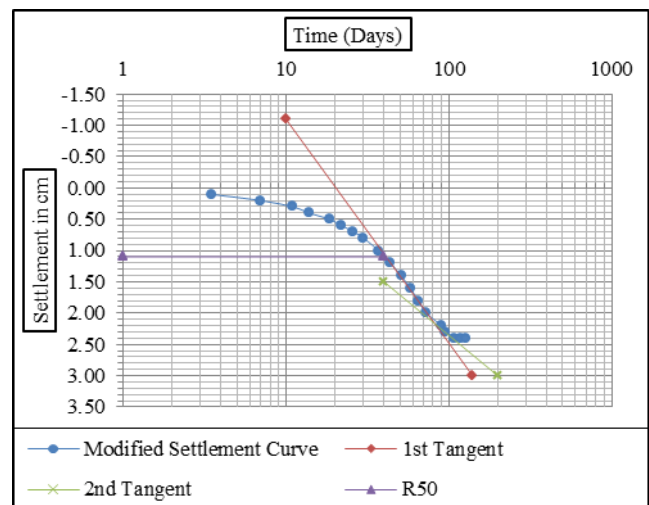


Fig. 9 Casagrande's Method (Settlement Marker) (CH-1)

• **Taylor's Method:**

$$C_{vz} = \frac{T_{90}}{t_{90}} H^2$$

Here, H = 500 cm, Tv = T₉₀ = 0.848, t = t₉₀ = 10454400 sec

$$c_{vz} = \frac{0.848 * (250)^2}{10454400}$$

Based on above parameters, $c_{vz} \text{ (field)} = 5.07 \times 10^{-03} \text{ cm}^2/\text{sec}$

• **Casagrande's Method:**

$$C_{vz} = \frac{T_{50}}{t_{50}} H^2$$

Here, H = 250 cm, Tv = T₅₀ = 0.197, t = t₅₀ = 3456000 sec

$$c_{vz} = \frac{0.197 * (250)^2}{3456000}$$

Based on above parameters, $c_{vz} \text{ (field)} = 3.56 \times 10^{-03} \text{ cm}^2/\text{sec}$

Based on the output generated after running the programme, relation is plotted on a log₁₀ – log₁₀ scale between coefficient of consolidation and time for 50%, 70% and 90% consolidation. On the log₁₀/log₁₀ scale, it is seen that for different spacing (S), these relations are all straight lines and parallel to one another. It is observed that, as c_v reduces (from 1 x 10⁰¹ cm²/sec to 1 x 10⁻⁰⁹ cm²/sec) the time increases at higher rate. As per the study of Curve Fitting/ Regression Analysis, the best fit curve by least square method is attempted. On taking log₁₀ c_v and log₁₀ t, the graph obtained was a set of

straight lines and hence the best fit curve to the obtained observations is $t=A \cdot C_v^B$. But from our analysis it is observed that, the B value works out to -1. Hence the equation reduces to, $t=A/C_v$. So it can be written as, $A=C_v \cdot t$.

For determining time for 90% consolidation of 11 m depth of clay layer, for $c_h = 1.0 c_v$ and for double drainage condition we get the following equation,

$$C_v \cdot t = 2.45E^{-03}S^2 + 6.69E^{-04}S - 5.04E^{-04}$$

Now, if we know any of two parameters from 1) coefficient of consolidation ' c_v ', 2) time ' t ' required for 50% consolidation, 70% consolidation or 90% consolidation or 3) spacing of vertical drain ' S ', we'll be able to determine the remaining parameter.

For example, in the project report of Virar case study, they have mentioned $c_{v\text{Laboratory}}$ to be taken as $2.30 \times 10^{-04} \text{ cm}^2/\text{sec}$, and Spacing taken for project is 1.50 m. By putting these values in the above equation, we get the time required for 90% consolidation is 240108.03 days.

C. Summary:

TABLE III COEFFICIENT OF CONSOLIDATION FIELD (C_{vz}) IN CM^2/SEC

Sr. No.	Marker	Chainage	Laboratory	Taylor's Method (c_{vt})	Casagrande's Method (c_{vc})
1	Settlement	CH-1/L1	2.57×10^{-04}	5.07×10^{-03}	3.56×10^{-03}
2	Settlement	CH-2/L1	2.57×10^{-04}	7.57×10^{-03}	5.94×10^{-03}
3	Settlement	CH-3/L1	2.57×10^{-04}	7.57×10^{-03}	5.70×10^{-03}
4	Settlement	CH-4/L1	2.25×10^{-04}	6.13×10^{-03}	4.07×10^{-03}
5	Settlement	CH-5/L1	2.25×10^{-04}	6.80×10^{-03}	4.75×10^{-03}
6	Piezometer	CH-1/L1	2.57×10^{-04}	7.92×10^{-03}	7.13×10^{-03}
7	Piezometer	CH-2/L1	2.57×10^{-04}	7.57×10^{-03}	6.20×10^{-03}
8	Piezometer	CH-3/L1	2.57×10^{-04}	9.58×10^{-03}	7.13×10^{-03}
9	Piezometer	CH-4/L1	2.25×10^{-04}	9.58×10^{-03}	7.13×10^{-03}
10	Piezometer	CH-5/L1	2.25×10^{-04}	9.83×10^{-03}	7.13×10^{-03}

TABLE IV. RATIO OF COEFFICIENT OF CONSOLIDATION BY TAYLOR'S & CASAGRANDE'S METHODS (C_{vt} , C_{vc}) TO COEFFICIENT OF CONSOLIDATION OF LAB ($C_{vz\text{LAB}}$)

Sr.No.	Marker	Chainage	Laboratory (C_{vlab})	Taylor's Method (C_{vt}/C_{vlab})	Casagrande's Method (C_{vc}/C_{vlab})
1	Settlement	CH-1/L1	2.57×10^{-04}	19.73	13.86
2	Settlement	CH-2/L1	2.57×10^{-04}	29.47	23.10
3	Settlement	CH-3/L1	2.57×10^{-04}	29.47	22.18
4	Settlement	CH-4/L1	2.25×10^{-04}	27.26	18.10
5	Settlement	CH-5/L1	2.25×10^{-04}	30.21	21.11
6	Piezometer	CH-1/L1	2.57×10^{-04}	30.82	27.72
7	Piezometer	CH-2/L1	2.57×10^{-04}	29.47	24.11
8	Piezometer	CH-3/L1	2.57×10^{-04}	37.29	27.72
9	Piezometer	CH-4/L1	2.25×10^{-04}	42.60	31.67
10	Piezometer	CH-5/L1	2.25×10^{-04}	43.68	31.67

So for the given property of soil, which are measured in laboratory multiplied by this ratio would give us the rate of consolidation which is likely to take place in the field. The same could be used as a multiplication factor with laboratory test data for all cases in this particular region to determine rate of consolidation, which is likely to occur thus giving us a brief idea of how the soil is going to be behaving under the stress conditions.

III. CONCLUSION

Thus, this data will be useful in planning of the given project i.e.

- In deciding the various factors such as time required for the stage loading,
- Time when the future activities can be started when a considerable amount of consolidation has taken place.
- Time taken to complete the project.
- Spacing required for the project to complete the consolidation process in desired time.

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